Problem Set #1

Deadline:28/09/2011

Problem(1)

In an experiment A, B, C and D are events with probabilities 0.2, 0.35, 0.625 and 0.375, respectively. A and B are disjoint while C and D are independent.
Find:

1. $P[A \cap B], P[A \cup B], P[A \cap B^C]$ and $P[A \cup B^C]$
2. Are A and B independent?
3. $P[C \cap D], P[C \cap D^C]$ and $P[C^C \cap D^C]$
4. Are C^C and D^C independent?

Problem(2)

Prove that:

$$P[AUBUC] = P[A] + P[B] + P[C] - P[A \cap B] - P[A \cap C] - P[B \cap C] + P[A \cap B \cap C]$$

Problem(3)

A box contains 7 red, 3 white balls. If a ball is drawn from the box a ball of the other color is then put into the box.
(a) Find the probability that the second ball is red?
(b) If both balls were of the same color what is the probability that they were both white?

Problem(4)

Find P[B/A] if:
(a) A is subset of B?
(b) A and B are mutually exclusive?

Problem(5)

In a certain colleague 25% of students failed in math, 15% failed in chemistry, 10% failed in both. A student is selected randomly.

(a) If he failed in chemistry what is the probability that he failed in math ?
(b) If he failed in math what is the probability that he failed in chemistry?
(c) Find probability that he failed in chemistry or math ?

Problem(6)

Suppose that X and Y are events and P[X]=0.4,P[Y]=0.6 , find P[X|Y] in the following cases:

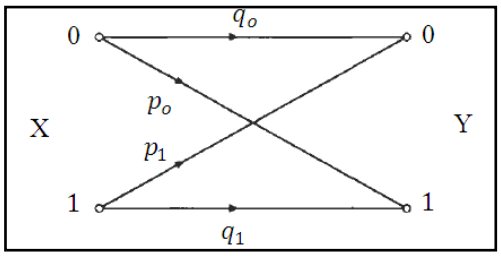
(a) X and Y are independent.
(b) X and Y are mutually exclusive.
(c) X is subset of Y
(d) Y is subset of X

Problem(7)

Consider the binary communication channel shown in Fig. The channel input symbol X may assume the state 0 or the state 1, and, similarly, the channel output symbol Y may assume either the state 0 or the state 1. Because of the channel noise, an input 0 may convert to an output 1 and vice versa. The channel is characterized by the channel transition probabilities p_o, q_o, p_1 *and* q_1 defined by:

$$p_o = P(y_1/x_o) \quad \text{and} \quad p_1 = P(y_o/x_1)$$
$$q_o = P(y_o/x_o) \quad \text{and} \quad q_1 = P(y_1/x_1)$$

where x_o and x_1 denote the events (X = 0) and (X = 1), respectively, and y_o and y_1 denote the events (Y = 0) and (Y = 1), respectively.



Note that $p_o + q_o = 1$, $p_1 + q_1 = 1$
Let $P(x_o) = 0.5$, $p_o = 0.1$, and $p_1 = 0.2$

(a) Find $P(y_o)$ *and* $P(y_1)$
(b) If a 0 was observed at the output, what is the probability that a 0 was the input state?
(c) If a 1 was observed at the output, what is the probability that a 1 was the input state?
(d) Calculate the probability of error Pe.